key

Algebra 3 Final Exam Review: Matrices

For #1-3, state the dimensions of the matrix.

1. $\begin{bmatrix} 1 & 4 & -6 \\ 2 & -3 & -3 \end{bmatrix}$
2×3

2. $\begin{bmatrix} 9 \\ -8 \\ 2 \end{bmatrix}$ 3×1

3. $\begin{bmatrix} -2 & 3 & -1 \\ 0 & 0 & 2 \\ -1 & 8 & 2 \end{bmatrix}$ 3×3

For #4-6, determine if each matrix product is possible. If so, state the dimensions of the product.

4. $A_{1\times5} \cdot B_{5\times7}$

yes; 1×7

5. $A_{2\times5} \cdot B_{2\times5}$

No

6. $A_{3\times2} \cdot B_{2\times4}$

yes; 3×4

For #7-9, determine the dimensions of matrix M.

7. $A_{1\times3} \cdot M = B_{1\times4}$

$M_{3\times4}$

8. $A_{2\times4} \cdot M = B_{2\times1}$

$M_{4\times1}$

9. $A_{3\times3} \cdot M = B_{3\times5}$

$M_{3\times5}$

For #10 -20, use matrices A-H.

$A = \begin{bmatrix} 1 & 4 & -6 \\ 2 & -3 & -3 \end{bmatrix}$
$B = \begin{bmatrix} 3 & 0 & 6 \\ 4 & -3 & 8 \end{bmatrix}$
$C = \begin{bmatrix} 5 & 2 \\ 2 & -1 \\ 5 & 4 \end{bmatrix}$
$D = \begin{bmatrix} -2 & 2 \\ 0 & -1 \\ -5 & 7 \end{bmatrix}$
$E = \begin{bmatrix} 2 & 4 \\ -3 & 1 \end{bmatrix}$

$F = \begin{bmatrix} -2 & 3 & -1 \\ 0 & 0 & 2 \\ -1 & 8 & 2 \end{bmatrix}$
$G = \begin{bmatrix} 5 & 3 & 0 \\ 0 & 4 & -1 \\ 3 & -5 & 0 \end{bmatrix}$
$H = \begin{bmatrix} 9 \\ -8 \\ 2 \end{bmatrix}$

10. $2A - \frac{1}{3}B$ $\begin{bmatrix} 1 & 8 & -14 \\ 2\frac{2}{3} & -5 & -8\frac{2}{3} \end{bmatrix}$

11. $C - D$ $\begin{bmatrix} 7 & 0 \\ 2 & 0 \\ 10 & -3 \end{bmatrix}$

12. $D \cdot A$ $\begin{bmatrix} 2 & -14 & 6 \\ -2 & 3 & 3 \\ 9 & -41 & 9 \end{bmatrix}$

13. E^{-1} (By hand) $\frac{1}{14}\begin{bmatrix} 1 & -4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{14} & \frac{-4}{14} \\ \frac{3}{14} & \frac{2}{14} \end{bmatrix}$

14. F^{-1} (By calculator) $\begin{bmatrix} \frac{-8}{13} & \frac{-7}{13} & \frac{3}{13} \\ \frac{-1}{13} & \frac{-5}{26} & \frac{2}{13} \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$

15. $-2F + G$ $\begin{bmatrix} 1 & 9 & -2 \\ 0 & 4 & 3 \\ -1 & 11 & 4 \end{bmatrix}$

16. $C \cdot D$

Not possible

17. $G \cdot H$ $\begin{bmatrix} 21 \\ -34 \\ 67 \end{bmatrix}$

18. $B \cdot D$ $\begin{bmatrix} 33 & -21 & 0 \\ 44 & -40 & 3 \end{bmatrix}$

19. Determinant of F
$\det([F]) = 26$

20. Determinant of E
14

21. Find AB

$A = \begin{bmatrix} 5 & 6 \\ 2 & 3 \\ -4 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 8 & 0 & -3 \\ 2 & 6 & 4 \end{bmatrix}$

$\begin{bmatrix} 52 & 36 & 9 \\ 22 & 18 & 6 \\ -32 & 0 & 12 \end{bmatrix}$

22. Find the determinant by hand of

$A = \begin{bmatrix} 5 & -3 & 2 \\ 6 & 10 & -1 \\ 1 & -2 & 0 \end{bmatrix}$

-51

For #23 – 26, solve the systems using matrices. Show the matrix equation you used.

23. $\begin{cases} -3x - 4y = 9 \\ 9x + 10y = -3 \end{cases}$ $X = 13$
    $y = -12$

24. $\begin{cases} x + 2y = 2 \\ 3x + 6y = 6 \end{cases}$

    Infinite solutions

25. $\begin{cases} 2x - y + 2z = 12 \\ x + 2y - 2z = -11 \\ 2x + y + 3z = 12 \end{cases}$ $X = 1$
    $y = -2$
    $z = 4$

26. $\begin{cases} -2x - 4z = 2 \\ -3y + 5z = -14 \\ x + 2y = 7 \end{cases}$ $X = 1$
    $y = 3$
    $z = -1$

Algebra 3 Final Exam Review: Chapter 3

For #1-4, rewrite each polynomial in standard form. Then identify the leading coefficient, degree and number of terms. Name each polynomial.

1. $7x^2 + 4x^5 - 3$ $4x^5 + 7x^2 - 3$
    Leading coefficient __4__
    Degree __5th__ Number of Terms __3__
    Name __Quintic trinomial__

2. $2 + 6x^3 + 2x - x$ $6x^3 + x + 2$
    Leading coefficient __6__
    Degree __3rd__ Number of Terms __3__
    Name __Cubic trinomial__

3. $10x - 3 - 2x^2 - 9x^3$ $-9x^3 - 2x^2 + 10x - 3$
    Leading coefficient __-9__
    Degree __3rd__ Number of Terms __4__
    Name __Cubic Tetranomial__

4. $-7x^2 - 4x$ $-7x^2 - 4x$
    Leading coefficient __-7__
    Degree __2nd__ Number of Terms __2__
    Name __Quadratic Binomial__

For #5-7, add or subtract. Write your answer in standard form.

5. $(4x^2 + 3) + (5x^2 + 4)$

$9x^2 + 7$

6. $(10x^3 - 7x^2) - (3x^3 + 2x^2 + 4)$

$7x^3 - 9x^2 - 4$

7. $(2 + 2x^4 - x) - (7 + x^4 + 3x)$

$x^4 - 4x - 5$

For #8-16, find each product.

8. $3y(2x^2 + 5xy)$

$6x^2y + 15xy^2$

9. $8(2x + 2)$

$16x + 16$

10. $(a + b)(4ab + b^2)$

$4a^2b + ab^2 + 4ab^2 + b^3$
$4a^2b + 5ab^2 + b^3$

11. $(8m + 2)(7m - 3)$

$56m^2 + -10m - 6$

12. $\left(3x + \frac{1}{4}\right)^2$

$9x^2 + \frac{3}{2}x + \frac{1}{16}$

13. $(3x - 2)(2x^3 - x^2 + 4x + 4)$

$6x^4 - 3x^3 + 12x^2 + 12x - 4x^3 + 2x^2 - 8x - 8$
$\boxed{6x^4 - 7x^3 + 14x^2 + 4x - 8}$

14. $(p^2 - p + 1)(6p^2 - 8p - 3)$

$6p^4 - 8p^3 - 3p^2 - 6p^3 + 8p^2 + 3p$
$+6p^2 + -8p - 3$ $\boxed{6p^4 - 14p^3 + 11p^2 - 5p - 3}$

15. $(b - 5)^3$

$b^3 + 75b + 75b - 125$

16. $(x + 3y)^3$

$x^3 + 9x^2y + 27y^2x + 27y^3$

For #17-24, factor each polynomial completely.

17. $3x^3 + 6x^2 + 5x + 10$

$= (3x^2 + 5)(x + 2)$

18. $21v^3 + 56v^2 - 12v - 32$

$(3v + 8)(7v^2 - 4)$

19. $12a^3 - 20a^2 + 21a - 35$

$(3a + -5)(4a^2 + 7)$

20. $21x^3 - 7x^2 + 6x - 2$

$(3x-1)(7x^2+2)$

21. $125 + 8u^3$

$(5+2u)(25-10u+4u^2)$

22. $374x^4 + 81x$

$x(374x^3+81)$

23. $125 - 27u^3$

$(5-3u)(25+15u+9u^2)$

24. $8x^4 + x$

$x(2x+1)(4x^2+2x+1)$

For #25-27, identify whether the function graphed has an even or odd degree, positive or negative leading coefficient and the end behavior of the function.

25.
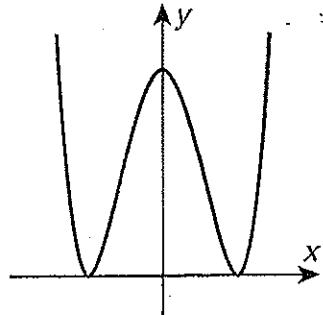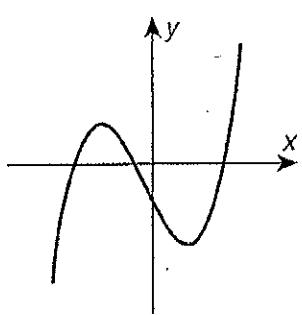
26.

27.

Odd or Even?

Negative or Positive?

As $x \to \infty, f(x) \to$ ___+∞___

As $x \to -\infty, f(x) \to$ ___+∞___

Odd or Even?

Negative or Positive?
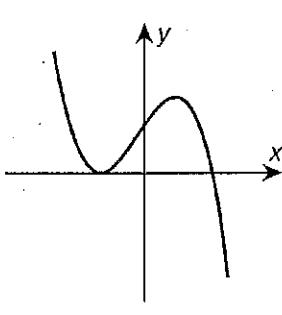
As $x \to \infty, f(x) \to$ ___+∞___

As $x \to -\infty, f(x) \to$ ___$-\infty$___

Odd or Even?

Negative or Positive?

As $x \to \infty, f(x) \to$ ___$-\infty$___

As $x \to -\infty, f(x) \to$ ___+∞___

Divide using long division

28. $(6x^2 + 7x - 2) \div (x + 4)$ $6x - 17 + \dfrac{66}{x+4}$

29. $(2x^2 - 9x + 10) \div (2x + 1)$ $x - 5 + \dfrac{15}{2x+1}$

Divide using synthetic division

30. $(3x^3 + 4x^2 - 8) \div (x - 2)$ $3x^2 + 10x + 20 + \dfrac{32}{x-2}$

31. $(2x^3 + 3x^2 - 6x - 4) \div (x - 1)$ $2x^2 + 5x - 1 + \dfrac{-5}{x-1}$

Identify the roots of each equation.

32. $x^3 + 15x^2 + 75x + 125 = 0$ -5 multiplicity 3

33. $x^3 - x^2 - 32x + 96 = 0$ No Rational roots

34. $8x^3 - 12x^2 + 6x - 1 = 0$ $\frac{1}{2}$ Multiplicity 3

35. $4x^3 + 16x^2 - 25x - 100 = 0$

$-4, \frac{5}{2}, \frac{-5}{2}$

Algebra 3 Final Exam Review: Chapter 4

Find the inverse of each function.

1. $f(x) = 15x$

$g(x) = \frac{x}{15}$

2. $f(x) = 3x + 2$

$g(x) = \frac{x-2}{3}$

3. $f(x) = 5 - \frac{3}{4}x$

$g(x) = \frac{-4}{3}(x - 5)$

Write each exponential equation in logarithmic form.

4. $3^5 = 243$

$\log_3 243 = 5$

5. $2^{-3} = \frac{1}{8}$

$\log_2 \frac{1}{8} = -3$

6. $16^{1.5} = 64$

$\log_{16} 64 = 1.5$

Write each logarithmic equation in exponential form.

7. $\log_{64} 512 = 1.5$

$64^{1.5} = 512$

8. $\log_2 8 = 3$

$2^3 = 8$

9. $\log_4 \frac{1}{16} = -2$

$4^{-2} = \frac{1}{16}$

Simplify each expression.

10. $\log_2 10 + \log_2 12.8$ $\log_2 128 = 7$

11. $\log_4 8 + \log_4 2$ $\log_4 16 = 2$

12. $\log_6 144 - \log_6 4$ $\log_6 \frac{144}{4} = 2$

13. $\log 10,000 - \log 100$ $\log \frac{10000}{100} = 2$

14. $\log_8 64^3 = 6$

15. $\log_3 3^{2x} = 2x$

Solve each equation.

16. $3^{x+1} = 9^4$ $x = 7$

17. $32^{x-2} = 8^x$ $x = 5$

18. $9^x = 12$ $x = \log_9 12$

19. $\log_6(4x - 9) = \log_6(x)$ $x = 3$

20. $\log_7(10x + 13) = 3$ $x = 33$

21. $\log(20x) - \log 4 = 2$ $x = 20$

22. $\log_9 x^3 = 8$ $x = \sqrt[3]{9^8}$

23. $\log x + \log(2x - 1) = 1$

$\log(2x^2 - x) = 1$ $2x^2 - x = 10$

$x = \text{(quad)} -2, 2.5$

24. $\log_3\left(\frac{2}{x}\right) + 2 = 0$ $x = 18$

Graph each function.

25. $f(x) = e^x - 1$
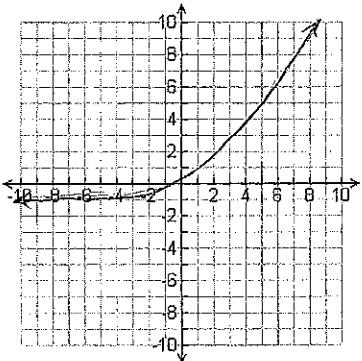
26. $f(x) = -\frac{1}{2}(3^x)$
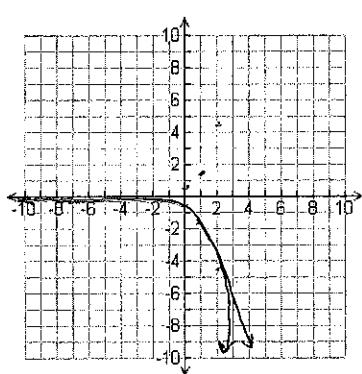
27. $f(x) = -4\log x$
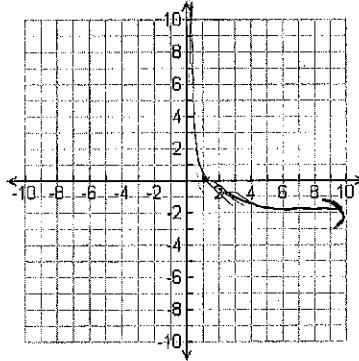
Algebra 3 Final Exam Review: Chapter 5

Simplify. Identify any x-values for which the expression is undefined.

1. $\frac{6x^3}{27x^2 + 12x}$ $x \neq 0, \frac{-4}{9}$ $\frac{2x}{9x + 4}$

2. $\frac{x^2 - x - 2}{3x - 6}$ $x \neq 2$ $\frac{x+1}{3}$

3. $\frac{-x^2 + 16}{-x^2 - 9x - 20}$ $x \neq 5, -4$ $\frac{x-4}{x+5}$

4. $\frac{4xy^3}{5x^2} \cdot \frac{20x^3y^2}{-16xy^7}$ $\frac{x}{-1y^2}$ $x \neq 0$ $y \neq 0$

5. $\frac{x^2 - 9}{2x + 10} \cdot \frac{x+5}{x-3}$ $x \neq 3, -5$ $\frac{x+3}{2}$

6. $\frac{x-4}{2x^2} \cdot \frac{x}{x^2 - x - 12}$ $x \neq 0, -3, 4$ $\frac{1}{2x(x+3)}$

7. $\frac{3x^3}{4x+4} \div \frac{9x}{x+1}$ $\frac{x^2}{12}$ $x \neq 1, 0$

8. $\frac{12x^3y^6}{9xy} \div \frac{6y^2}{3x}$ $\frac{2x^3y^3}{3}$ $x \neq 0$ $y \neq 0$

9. $\frac{x^2 - 16}{x^2 + 4x + 3} \div \frac{x-4}{x+1}$ $x \neq -1, -3, 4$ $\frac{x+4}{x+3}$

Add or Subtract. Identify any x-values for which the expression is undefined.

10. $\frac{x+9}{2x+1} + \frac{3x+6}{2x+1} = \frac{4x+15}{2x+1}$ $x \neq \frac{-1}{2}$

11. $\frac{2}{x+3} + \frac{4x}{x^2 - 9} = \frac{6}{x+3}$ $x \neq 3, -3$

12. $\frac{1}{x^2 + 6x + 8} + \frac{1}{x^2 - 6x - 16} = \frac{2x+4}{(x+2)(x-8)(x+4)}$ $x \neq -2, 8, -4$

13. $\frac{x-6}{x+5} - \frac{8x+7}{x+5} = \frac{-7x - 13}{x+5}$ $x \neq 5$

14. $\frac{x}{x+1} - \frac{3}{x+4} = \frac{x^2 + x - 3}{(x+1)(x+4)}$ $x \neq -1, -4$

15. $\frac{7}{x-9} - \frac{2x-6}{x^2 - 13x + 36} = \frac{5x - 22}{x^2 - 13x + 36}$ $x \neq 4, 9$

Simplify.

16. $\frac{\frac{3x}{3x+21}}{\frac{9x^2}{x+7}} = \frac{1}{9x}$

$x \neq -7, 0$

17. $\frac{\frac{x}{x-1}}{\frac{10x^2}{-4x+4}} = \frac{-2}{5x}$

$x \neq 1, 0,$

18. $\frac{\frac{1}{x-2}}{\frac{x+3}{x^2 - 4}} = \frac{x+2}{x+3}$

$x \neq 2, -2, -3$

Using the graph of $f(x) = \frac{1}{x}$ as a guide, describe the transformation and graph each function.

19. $g(x) = \frac{1}{x-4}$
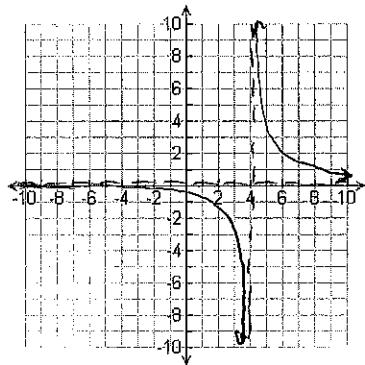
20. $g(x) = \frac{1}{x} + 6$
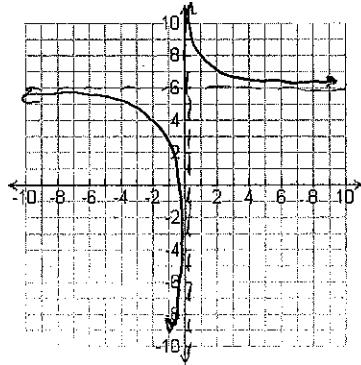
21. $g(x) = \frac{1}{x+2} - 5$
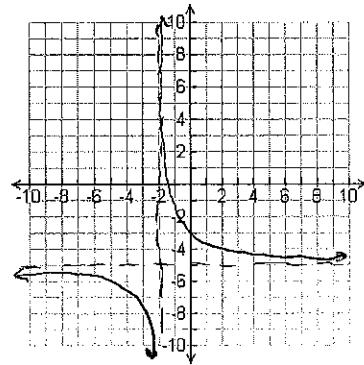
Identify the zeros, asymptotes and holes of each function. Then graph each function.

22. $f(x) = \frac{x^2-5x-24}{2x+1} = \frac{(x-8)(x+3)}{2x+1}$

23. $f(x) = \frac{2x^2-3x-2}{x-4} =$

$Z : (8,0)\ (-3,0)$

$SA: y=\frac{1}{2}x+\frac{9}{4}$

$H: N/A$

$VA: x = -\frac{1}{2}$
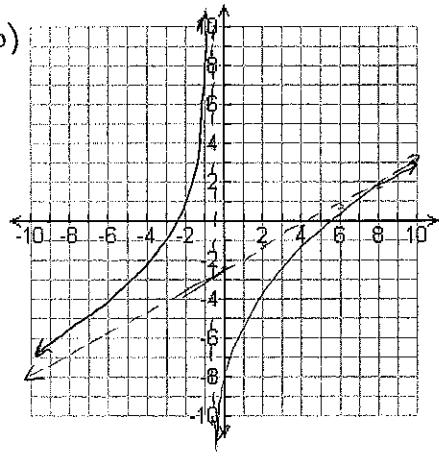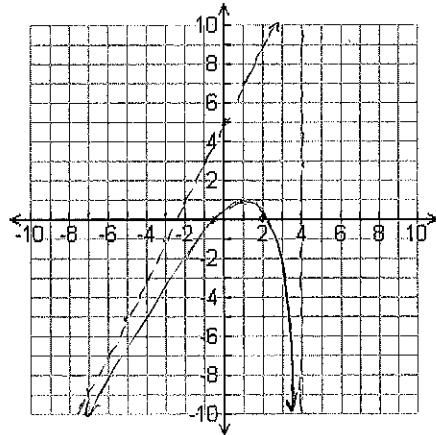
$Z: (2,0)\ (-.5,0)$

$SA: y = 2x+5$

$H: N/A$

$VA: x = 4$

$y\ int : (0,.5)$

24. $f(x) = \frac{-3x^2+8x-4}{x^2-25}$

25. $f(x) = \frac{x^2-4x-21}{x+3} = \frac{(x-7)(x+3)}{}$

$HA: y=-3$

$VA : x=5, -5$

$Z: (2,0)(\frac{2}{3},0)$

$H: N/A$
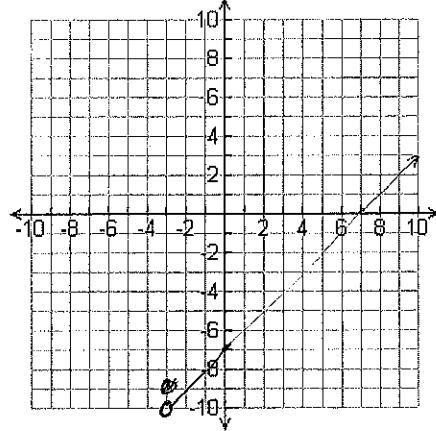
$H: x=-3\ (-3,-10)$

$SA: y = x-7$

$VA: N/A$

$Z: (7,0)$

26. $f(x) = \dfrac{x^2-4x-5}{x^2-25} = \dfrac{(x-5)(x+1)}{(x-5)(x+5)}$

27. $f(x) = \dfrac{x^2-3x}{4x-12} = \dfrac{x(x-3)}{4(x-3)}$

H: $(5, \frac{3}{5})$
HA: $y = 1$
VA: $x = -5$
$t \neq (-1, 0)$

H: $(3, \frac{3}{4})$
SA: $y = \frac{x}{4}$
HA: N/A
$z: (0, 0)$
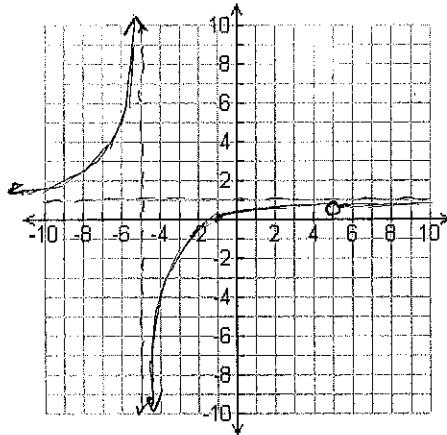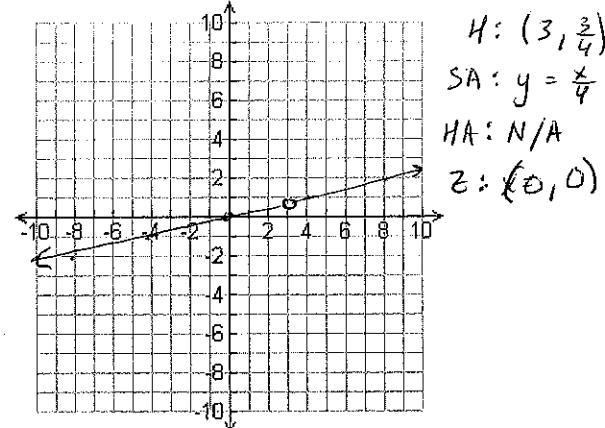
Solve each equation.

28. $12 + \dfrac{2}{3x} = 6$ $x = \dfrac{-1}{9}$

29. $x - \dfrac{1}{x} = \dfrac{35}{x}$ $x = \pm 6$

30. $\dfrac{x}{x+1} + \dfrac{x}{4} = \dfrac{3x}{4x+4}$ $x = 0, -2$

31. $\dfrac{x-1}{x-4} = \dfrac{x+6}{x}$ $x = 8$

32. $\dfrac{6x}{x+5} = \dfrac{2x-20}{x+5}$ $x = -5$ extraneous   (No solution)

33. $\dfrac{4}{x-4} = -\dfrac{x}{x-4} + \dfrac{x}{2}$ $x = 3 \pm \sqrt{17}$

Solve each inequality

34. $\dfrac{2x+1}{x} \geq 3$ $(0, 1]$

35. $\dfrac{10}{x-2} < 2$ $(-\infty, -2) \cup [7, \infty)$

36. $\dfrac{15}{x+3} \leq 1$ $(-\infty, -3) \cup [12, \infty)$

Graph each function. Identify the domain and range.

37. $f(x) = \sqrt{x-4} + 1$

D: $x \geq 4$
R: $y \geq 1$

38. $f(x) = -\dfrac{1}{2}\sqrt{x}$

D: $x \geq 0$
R: $y \leq 0$

39. $f(x) = 2\sqrt[3]{x+2}$
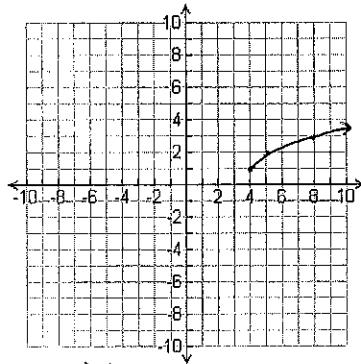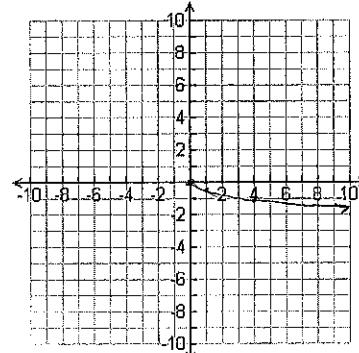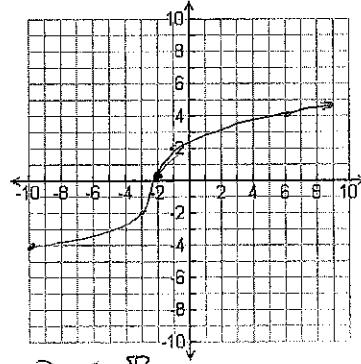
D: $x = \mathbb{R}$
R: $y = \mathbb{R}$

Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformations.

40. $g(x) = \sqrt{x-8}$
Right 8

41. $g(x) = -6\sqrt{x}$
Reflection over x axis
Vertical stretch

42. $g(x) = \dfrac{1}{3}\sqrt{x} + 2$
Vertical Compression
up 2

Algebra 3 Final Exam Review: Chapter 10

Use a trig function to solve for x.

1.
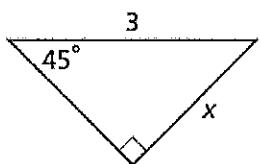
$\sin(45) = \frac{x}{3}$

$3 \cdot \sin(45) = x$

$3 \cdot \frac{\sqrt{2}}{2} = x$

$\boxed{x = \frac{3\sqrt{2}}{2}}$

2.
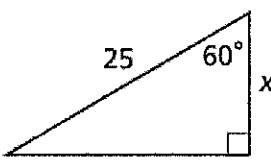
$\cos 60 = \frac{x}{25}$

$25 \cdot \cos 60 = x$

$25 \cdot \frac{1}{2} = x$

$x = \frac{25}{2}$

Find the values of the six trig functions of Θ.

3.

$\sin\theta = \frac{5\sqrt{34}}{34}$ $\csc\theta = \frac{\sqrt{34}}{5}$

$\cos\theta = \frac{3\sqrt{34}}{34}$ $\sec\theta = \frac{\sqrt{34}}{3}$

$\tan\theta = \frac{5}{3}$ $\cot\theta = \frac{3}{5}$

$5^2 + b^2 = 34 \quad b = 3$

4.
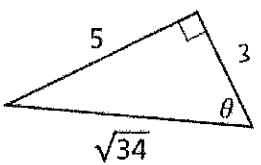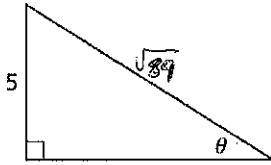
$\sin\theta = \frac{5\sqrt{89}}{89}$ $\csc\theta = \frac{\sqrt{89}}{5}$

$\cos\theta = \frac{8\sqrt{89}}{89}$ $\sec\theta = \frac{\sqrt{89}}{8}$

$\tan\theta = \frac{5}{8}$ $\cot\theta = \frac{8}{5}$

Find the measures of a positive and a negative angle that are coterminal with the given angle.

5. $\theta = 20°$

$380°, -340°$

6. $\theta = 400°$

$40°, -320°$

7. $\theta = -125°$

$-485, 235°$

Find the measure of the reference angle for each given angle.

8. $\theta = -120°$

$\theta_R = 60°$

9. $\theta = 175°$

$\theta_R = 5°$

10. $\theta = 110°$

$\theta_R = 70°$

P is a point on the terminal side of θ in the standard position. Find the exact value of the all six trig functions for Θ.

11. $P(2, 3)$ $\sin\theta = \frac{3\sqrt{13}}{13}$ $\csc\theta = \frac{\sqrt{13}}{3}$

$\cos\theta = \frac{2\sqrt{13}}{13}$ $\sec\theta = \frac{\sqrt{13}}{2}$

$\tan\theta = \frac{3}{2}$ $\cot\theta = \frac{2}{3}$

12. $P(-1, 4)$ $\sin\theta = \frac{4\sqrt{17}}{17}$ $\csc\theta = \frac{\sqrt{17}}{4}$

$\cos\theta = \frac{-1\sqrt{17}}{17}$ $\sec\theta = \frac{\sqrt{17}}{-1}$

$\tan\theta = \frac{4}{-1}$ $\cot\theta = \frac{-1}{4}$

Covert from degrees to radians or radians to degrees.

13. $-125° \; \frac{-25\pi}{36}$

14. $10° \; \frac{\pi}{18}$

15. $\frac{\pi}{10} = 18°$

16. $-\frac{\pi}{18} = -10°$ or $350°$

Use the unit circle to find the exact value of each trig function.

17. $\cos 150° = -\frac{\sqrt{3}}{2}$

18. $\tan\frac{7\pi}{4} = -1$

19. $\sin\frac{7\pi}{6} = \frac{-1}{2}$

20. $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

21. $\cos 270° = 0$

22. $\csc 225° = -\sqrt{2}$

**Find the area of each triangle. Round to the nearest tenth.**

23.

3 in.

70°

5 in.
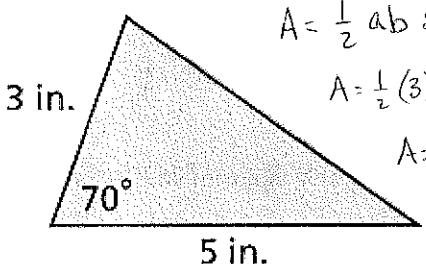
$A = \frac{1}{2} ab \sin C$

$A = \frac{1}{2}(3)(5) \sin(70°)$

$A = 7.047 \text{ units } (in^2)$

24.

7.1 ft

35°

6.2 ft
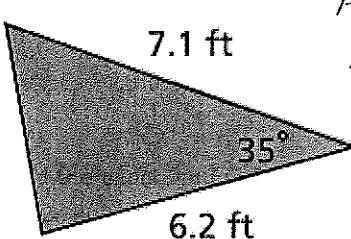
$A = \frac{1}{2} ab \sin C$

$A = \frac{1}{2}(7.1)(6.2) \sin(35°)$

$A = 12.6244 \text{ units } (ft^2)$

Solve each triangle. Round to the nearest tenth.

25.

P

3.73729 $83°$ 6.3917

q r

$65°$ $32°$

R 7 Q

$\dfrac{7}{\sin 83} = \dfrac{r}{\sin 65°} = \dfrac{q}{\sin 32°}$

26.

L

$52°$ $k = 24.9947$

21.2857

j

$57°$ J

$80°$

K 20

$\dfrac{20}{\sin 52} = \dfrac{k}{\sin 80} = \dfrac{j}{\sin 57}$

27.

3.8917

B

c

$29°$

A $20°$ $a = 1.7636$

$131°$

2.5

C

$\dfrac{2.5}{\sin 29°} = \dfrac{c}{\sin 131°} = \dfrac{a}{\sin 20°}$

28.

C

74.06

9 $a = 7.4807$

$46°$ 59.93

A 10 B
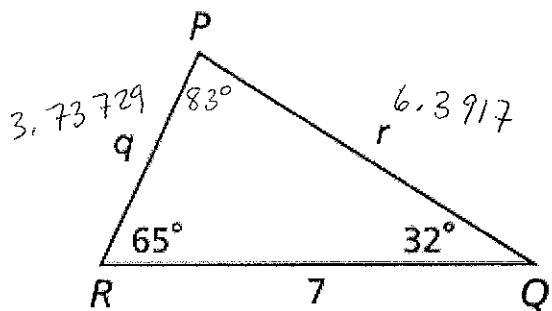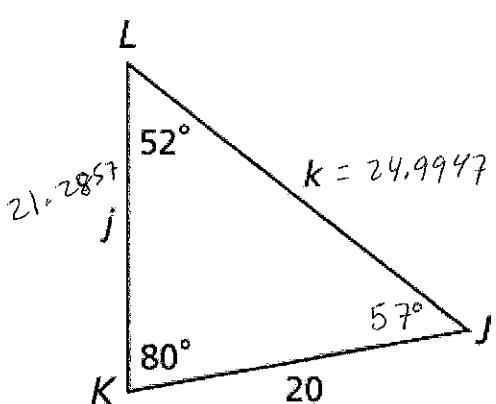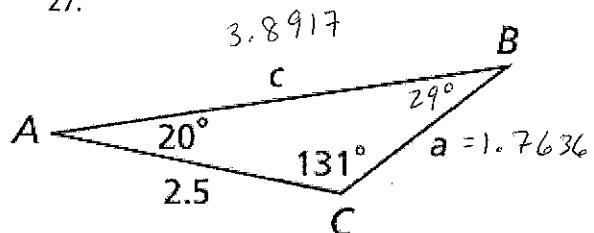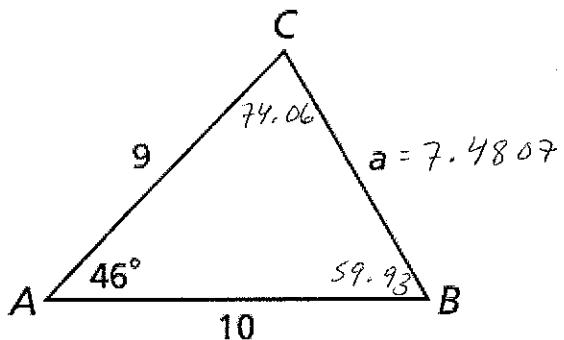
$a^2 = 9^2 + 10^2 + 2(9)(10)\cos(46°)$

$\dfrac{\sin(46)}{7.4807} = \dfrac{\sin B}{9}$

29.

7.0157 M

n $74.004°$

L $38.995°$

5

7.8 $62°$

N
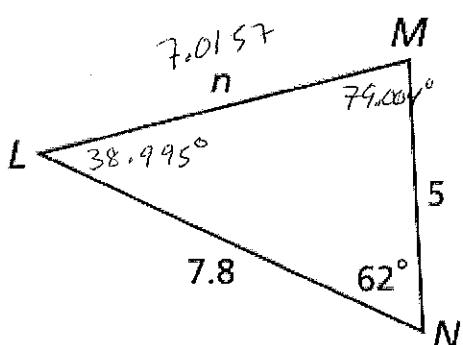
$n^2 = 5^2 + 7.8^2 - 2(5)(7.8)\cos(62)$

$\dfrac{\sin 62}{7.0157} = \dfrac{\sin(M)}{7.8}$ $M = 79.004$

30.

E

18.886 $d = 12.1929$

10.1 51.113 F

$110°$ 4.2

D
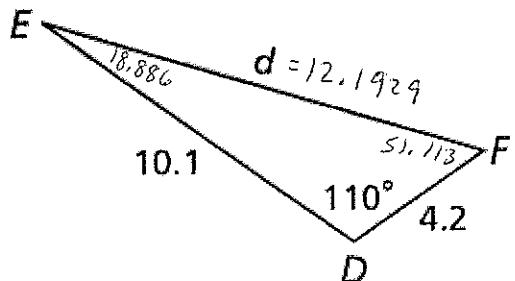
$d^2 = 10.1^2 + 4.2^2 - 2(4.2)(10.1)\cos(110°)$

$\dfrac{\sin 110}{12.1929} = \dfrac{\sin E}{4.2}$ $E = 18.886°$